10.2 Separation of Variables How to solve a class of differential equations of the form y' = p(t)q(y)Called "seperable" because me can seperate the variables $y' = \frac{3t^2}{y^2} \implies \boxed{y^2y' = 3t^2}$ y's on one side, is on the other. p(t)=5 seperable => 1/= 5 e^{x} y' = 5ysuperable =) y=3t+4 p q(y)=1 (2) y = 3t+ 4 (3) y'= e'(2t+1) seperable -) e'y'= 2t+1 (4) y' = 5y - 2tnot seperably connet write 5y-2t as plt)g(y)

How to find solutions: $y' = \frac{3t^2}{y^2} \implies y^2 y' = 3t^2$ write y' as dy de $\gamma^2 \frac{dx}{dt} = 3t^2$ $\int y^2 \frac{dy}{dt} dt = \int 3t' dt$ not exactly Canceling, but we can think of it that way. for simplicity. $\int y^2 dy = \int 3t^2 dt$ $\frac{1}{3}y^{3} + C_{1} = t^{3} + C_{2}$ $\frac{1}{3}y^{3} = t^{3} + (c_{2} - c_{i})$ 62-6, is just a constant. $\frac{1}{3}y^3 = t^3 + C_3$ so only need + C on one side y3= 3t + 3C3 since 36s is stall or constant, we can just rewrite it as L $y^{3} = 3t^{3} + C_{M}$ $y = \sqrt[3]{3t^3} + C$

 $y' = t^3 y^2 + y^2$ y'= (t2+1) y 2 seperable $\frac{1}{y^2}\frac{dy}{dt} = t^3 + 1$ $\int \frac{1}{y^2} dy = \int t^3 t | dt$ $-y' = \frac{t^{4}}{34} + t + C$ this give does $\gamma = \frac{-1}{\frac{1}{4}t^{4}+t+c}$ a/1 the colutions? no! (onsider y=0 check: 1=0 シ 0= t3.0+0 @ V

Solve the IVP:

$$y' = e^{\gamma}(2t+1) \quad y(0) = 1$$

$$e^{\gamma}y' = 2t+1$$

$$\int e^{\gamma}dy = (2t+1)dt$$

$$e^{\gamma} = t^{2}+t+C$$

$$y = \ln(t^{2}+t+C)$$
using $y(0) = 1$

$$M(t^{2}+t+C)$$

$$1 = \ln(C)$$

$$= 2 e = C$$

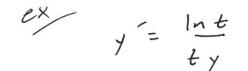
$$y = \ln(t^{2}+t+e)$$

 $\int \frac{dy}{7} = \int k dt$ In(y) = KttC y = e kt+c y = Cekt

-1'= KY

ex

y = K(12-y) $\frac{y}{12-y} = k$ $\int \frac{1}{12-y} dy = \int k dt$ 1-112-y] = kt+C 12-7 = Cekt y= 12-6ekt



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dy = tet

use integration by parts

 $e^{\chi} = (\gamma - 3)^2 \ln t$

This is also integration by parts. Hint: u = ln(x), dv = 1

 $\frac{dY}{dt} = \frac{t^2 \gamma^2}{\frac{t^3 + 8}{t^3 + 8}}$

$$\frac{dy}{dx} = \frac{\ln x}{\sqrt{x_y}}, \quad y(1) = 4$$
 use integration
 $\frac{dy}{dx} = \frac{\ln x}{\sqrt{x_y}}, \quad y(1) = 4$ by parts

 $ex = y^2 - e^{3t} + (0) = 1$